Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

■ : explanation
∵ is 'because'
∴ is 'therefore'

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA11/01**

Mathematics

October 2022

International Advanced Subsidiary/Advanced Level

Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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B:1/1/1/1/

P 7 2 1 3 7 A 0 1 2 8

Pearson

1. The curve C has equation

$$y = \frac{x^3}{4} - x^2 + \frac{17}{x} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(3)

The point $R\left(2, \frac{13}{2}\right)$ lies on C.

(b) Find the equation of the tangent to C at the point R. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

a) dy/dx is differential of y.

① rewrite equation in easier form for integration.

$y = \frac{1x^3}{4} - x^2 + \frac{17}{x^1} = \frac{1}{4}x^3 - x^2 + \frac{17}{x^1}{}^* = \frac{1}{4}x^3 - x^2 + 17x^{-1}$

* indicies rule : $\frac{a}{x^b} = ax^{-b}$

② differentiation

$\frac{dy}{dx} = 3\left(\frac{1}{4}x^{3-1}\right) + 2\left(-x^{2-1}\right) + (-1)\left(17x^{-1-1}\right)$

$= \frac{3}{4}x^2 - 2x^1 - 17x^{-2}$

$\therefore \quad \frac{dy}{dx} = \frac{3}{4}x^2 - 2x - 17x^{-2}$

b) tangent means gradient of tangent is same as gradient of equation

① to find gradient of tangent, substitute x-value of R into dy/dx (the gradient function) of curve C.

$\left.\frac{dy}{dx}\right|_{x=2} = \frac{3}{4}(2)^2 - 2(2) - 17(2)^{-2} = \frac{12}{4} - 4 - \frac{17}{4} = -\frac{21}{4}$

Question 1 continued

(2) Find equation of tangent using line passing through (a, b) and gradient M

equation: $(y-b) = M(x-a)$

$a = 2$
$b = 13/2$
$M = \frac{-21}{4}$

$(y - 13/2) = -21/4(x - 2)$

(3) write equation in the form $ax + by + c = 0$

$y - 13/2 = -21/4(x-2)$
$y - 13/2 = -21/4\,x + 21/2$

$\times 4$ $\quad$ $\times 4$

$4y - 26 = -21x + 42$

$+21x$ $\quad$ $+21x$

$21x + 4y - 26 = 42$

-42 $\quad$ -42

$21x + 4y - 68 = 0$

$\therefore\ 21x + 4y - 68 = 0$

(Total for Question 1 is 6 marks)

P 7 2 1 3 7 A 0 3 2 8

2. Given that

means "identical to"

$$(x-5)(2x+1)(x+3) \equiv ax^3 + bx^2 - 32x - 15$$

where a and b are constants,

(a) find the value of a and the value of b.

(2)

(b) Hence find

$$\int \frac{(x-5)(2x+1)(x+3)}{5\sqrt{x}}\,dx$$

writing each term in simplest form.

(5)

a) expanding brackets

$(x-5)(2x+1)(x+3) = (2x^2+x-10x-5)(x+3) = (2x^2-9x-5)(x+3)$

$(2x^2-9x-5)(x+3) = (2x^3+6x^2-9x^2-27x-5x-15)$

$= 2x^3-3x^2-32x-15$

$\therefore\ (x-5)(2x+1)(x+3) \equiv 2x^3-3x^2-32x-15$

$\therefore\ a=2 \quad b=-3$

b) $\displaystyle\int \frac{(x-5)(2x+1)(x+3)}{5\sqrt{x}}\,dx = \int \frac{(2x^3-3x^2-32x-15)}{5\sqrt{x}}\,dx$ from part (a)

② Write in easier form for integration

$\dfrac{2x^3-3x^2-32x-15}{5\sqrt{x}} = \dfrac{2x^3-3x^2-32x-15}{5x^{1/2}}$ ① ① indicies rule: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$\because x^0 = 1$

$= \dfrac{2x^3}{5\,x^{1/2}} - \dfrac{3x^2}{5\,x^{1/2}} - \dfrac{32x^1}{5\,x^{1/2}} - \dfrac{15x^0}{5\,x^{1/2}} = \dfrac{2}{5}\dfrac{x^3}{x^{1/2}} - \dfrac{3}{5}\dfrac{x^2}{x^{1/2}} - \dfrac{32}{5}\dfrac{x^1}{x^{1/2}} - \dfrac{15}{5}\dfrac{x^0}{x^{1/2}}$

② $= \dfrac{2}{5}x^{3-1/2} - \dfrac{3}{5}x^{2-1/2} - \dfrac{32}{5}x^{1-1/2} - 3x^{0-1/2}$

② indicies rule $\dfrac{a^b}{a^c} = a^{b-c}$

P 7 2 1 3 7 A 0 4 2 8

Question 2 continued

$$= \frac{2}{5}x^{5/2} - \frac{3}{5}x^{3/2} - \frac{32}{5}x^{1/2} - 3x^{-1/2}$$

3 integrate

$$\int \frac{2}{5}x^{5/2} - \frac{3}{5}x^{3/2} - \frac{32}{5}x^{1/2} - 3x^{-1/2}\,dx$$

$$= \left[\left(\frac{2/5}{5/2+1}x^{5/2+1}\right) + \left(\frac{-3/5}{3/2+1}x^{3/2+1}\right) + \left(\frac{-32/5}{1/2+1}x^{1/2+1}\right) + \left(\frac{-3}{-1/2+1}x^{-1/2+1}\right)\right]$$

$$= \frac{4}{35}x^{7/2} - \frac{6}{25}x^{5/2} - \frac{64}{15}x^{3/2} - 6x^{1/2} + C$$

DON'T FORGET OR WILL LOSE A MARK !!!

$$\therefore \frac{4}{35}x^{7/2} - \frac{6}{25}x^{5/2} - \frac{64}{15}x^{3/2} - 6x^{1/2} + C$$

(Total for Question 2 is 7 marks)

3. The share price of a company is monitored.

Exactly 3 years after monitoring began, the share price was £1.05

Exactly 5 years after monitoring began, the share price was £1.65

The share price, £V, of the company is modelled by the equation

[check units]

$$V = pt + q$$

where t is the number of years after monitoring began and p and q are constants.

(a) Find the value of p and the value of q.

(3)

Exactly T years after monitoring began, the share price was £2.50

(b) Find the value of T, according to the model, giving your answer to one decimal place.

(2)

a) form simultaneous equations with information given

① 3 years, £1.05

$t = 3 \quad V = 1.05 \quad \Rightarrow 1.05 = p3 + q$

② 5 years, £1.65

$t = 5 \quad V = 1.65 \quad \Rightarrow 1.65 = p5 + q$

Solve equation:

$1.65 = 5p + q$

$1.05 = 3p + q \quad -$

$0.60 = 2p$

$\therefore 2p = 0.60$

$\div 2 \quad p = 0.30 \quad \div 2$

Substitute p to solve for q.

$1.05 = 3p + q$

$1.05 = 3(0.30) + q$

$1.05 = 0.90 + q$

$-0.90 \quad 0.15 = q \quad -0.90$

$\therefore \quad p = 0.30 \qquad q = 0.15$

b) ① form equation. £V = pt + q

$V = 2.50$ given in question

$p = 0.30$ from part (a)

$t = T$ in question

$q = 1.50$ from part (a)

$\therefore\ 2.50 = 0.30T + 1.50$

② Solve for T.

$$2.50 = 0.3T + 0.15$$

$$-0.15 \quad 2.35 = 0.3T \quad -0.15$$

$$\div 0.3 \quad \frac{47}{6} = T \quad \div 0.3$$

$T = 47/6 = 7.83\dot{3}$

$\therefore\ T = 7.8$ (1dp)

(Total for Question 3 is 5 marks)

4. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

$$f(x) = x^2(2x + 1) - 15x$$

(a) Solve

$$f(x) = 0$$

(4)

(b) Hence solve

$$y^{\frac{4}{3}}\left(2y^{\frac{2}{3}} + 1\right) - 15y^{\frac{2}{3}} = 0 \qquad y > 0$$

giving your answer in simplified surd form.

(2)

a) $f(x) = \underbrace{x^2(2x+1)}_{} - 15x = 0$

$\underbrace{2x^3 + x^2} - 15x = 0$

Factorise : $x(2x^2 + x - 15) = 0$

$x(2x-5)(x+3) = 0$

Solve : $x_1 = 0$ $\quad 2x - 5 = 0 \quad 2x = 5 \quad x_2 = 5/2$ $\quad x + 3 = 0 \quad x_3 = -3$

$\therefore x = -3, 0, \frac{5}{2}$

b) let $x = y^{\frac{2}{3}}$

$\Rightarrow y^{\frac{4}{3}} = y^{\frac{2}{3}\times 2} = \left(y^{\frac{2}{3}}\right)^2 = (x)^2 = x^2$

so $y^{4/3}(2y^{2/3} + 1) - 15y^{2/3} = 0$ can be written as $x^2(2x+1) - 15x = 0$

Solve (using answers from part a)

$x_1 = 0 \quad x_2 = 5/2 \quad x_3 = -3$

$y^{2/3} = 0 \quad y^{2/3} = 5/2 \quad y^{2/3} = -3$

Question 4 continued

$y^{2/3}$ can be written as $\sqrt[3]{y^2}$ $\because$ indicies rule: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$\therefore$ ① $\sqrt[3]{y^2} \neq 0$ $\because$ $y > 0$

② $\sqrt[3]{y^2} = 5/2$

cube $(\sqrt[3]{y^2})^3 = \left(\frac{5}{2}\right)^3$ cube

$y^2 = \frac{125}{8}$

square root $y = \sqrt{\frac{125}{8}}$ square root

$y = \frac{\sqrt{125}}{\sqrt{8}} = \frac{\sqrt{25 \times 5}}{\sqrt{4 \times 2}} = \frac{\sqrt{25}\sqrt{5}}{\sqrt{4}\sqrt{2}} = \frac{5\sqrt{5}}{2\sqrt{2}}$

rationalising the surd : $\frac{5\sqrt{5}}{2\sqrt{2}} \frac{\times 2\sqrt{2}}{\times 2\sqrt{2}} = \frac{10\sqrt{10}}{8} = \frac{10}{8}\sqrt{10} = \frac{5}{4}\sqrt{10}$

③ $\sqrt[3]{y^2} = -3$

cube $(\sqrt[3]{y^2})^3 = (-3)^3$ cube

$y^2 \neq -27$

UNDEFINED $\because$ cannot have a negative square number

$\therefore$ $y = \frac{5}{4}\sqrt{10}$

(Total for Question 4 is 6 marks)

P 7 2 1 3 7 A 0 9 2 8

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \mathrm{f}(x)$, $x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{12}{\sqrt{x}} + \dfrac{x}{3} - 4$
- the point $P(9, 8)$ lies on C

(a) find, in simplest form, $\mathrm{f}(x)$

(5)

The line l is the normal to C at P

(b) Find the coordinates of the point at which l crosses the y-axis.

(4)

a) $f(x) \xrightarrow{\text{differentiate}} f'(x)$, $f'(x) \xrightarrow{\text{integrate}} f(x)$

① rewrite $f'(x)$ in easier form for integration

$f'(x) = \dfrac{12}{\sqrt{x}} + \dfrac{1x}{3} - 4 = \dfrac{12}{\sqrt{x}} + \dfrac{1}{3}x - 4 = \dfrac{12}{x^{1/2}} + \dfrac{1}{3}x - 4$

① indicies rule $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$= \dfrac{12}{x^{1/2}} + \dfrac{1}{3}x - 4 = 12x^{-1/2} + \dfrac{1}{3}x - 4$

② indicies rule : $\dfrac{a}{x^b} = ax^{-b}$

② integrate

$f(x) = \int f'(x)\,dx = \int 12x^{-1/2} + \frac{1}{3}x^1 - 4x^0\,dx$ $\quad \because x^0 = 1$

$= \left[\left(\dfrac{12}{-\frac{1}{2}+1} x^{-1/2+1}\right) + \left(\dfrac{1/3}{1+1} x^{1+1}\right) + \left(\dfrac{-4}{0+1} x^{0+1}\right)\right]$

$= 24x^{1/2} + \dfrac{1}{6}x^2 - 4x + C$

③ find $+C$ by substituting $P(9,8)$

$f(9) = 24(9)^{1/2} + 1/6(9)^2 - 4(9) + C = 8$

$= 72 + 81/6 - 36 + C = 8$

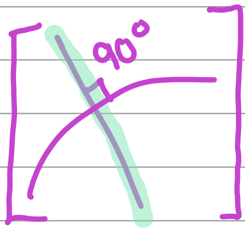
$= 99/2 + C = 8$

$-\frac{99}{2}$ $\quad C = -83/2 \quad$ $-\frac{99}{2}$

Question 5 continued

$\therefore \quad f(x) = 24x^{1/2} + \frac{1}{6}x^2 - 4x - \frac{83}{2}$

b) normal is Perpendicular to curve



$\therefore$ we find gradient of normal (m_n) using Perpendicular gradient rule $m_{normal} \times m_{curve} = -1$

We want to find equation of normal l so we can find when the line intersects with y-axis.

① find gradient of curve at point $P(9, 8)$

$f'(9) = \frac{12}{\sqrt{9}} + \frac{9}{3} - 4 = 4 + 3 - 4 = 3$

② find gradient of normal using formula.

$$m_n \times m_c = -1$$

$$\div 3 \curvearrowleft \quad m_n \times 3 = -1 \quad \curvearrowright \div 3$$

$$m_n = -1/3$$

③ find equation of normal using line passing through (a, b) and gradient m

equation: $(y - b) = m(x - a)$

$a = 9$
$b = 8$
$m = -1/3$

$$(y - 8) = -1/3(x - 9)$$

④ to find intersection of y-axis with line l, substitute $x = 0$.

$$y - 8 = -1/3(0 - 9)$$

$$+8 \curvearrowleft \quad y - 8 = 3 \quad \curvearrowright +8$$

$$y = 11$$

$\therefore$ line l intersects y-axis at $(0, 11)$

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

P 7 2 1 3 7 A 0 1 3 2 8

6. (a) Given that k is a positive constant such that $0 < k < 4$ sketch, on separate axes, the graphs of

(i) $y = (2x - k)(x + 4)^2$

(ii) $y = \dfrac{k}{x^2}$

showing the coordinates of any points where the graphs cross or meet the coordinate axes, leaving coordinates in terms of k, where appropriate.

(5)

(b) State, with a reason, the number of roots of the equation

$$(2x - k)(x + 4)^2 = \frac{k}{x^2}$$

(1)

Working out:

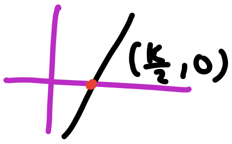
i) $y = (2x-k)(x+4)^2$ ① $(\ldots x \ldots)(x \ldots)^2$ indicates cubic graph as x^3 value present in expanded equation. So shape is ∿ or ∿.

② $(+2x \ldots)(+x \ldots) = +2x^3 + \ldots$
x^3 value in expanded bracket is positive ∴ Shape of cubic graph is ∿

③ When $y = 0$ $(2x-k)(x+4)^2 = 0$

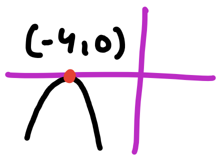
① $2x - k = 0$
$2x = k$
$x = \frac{k}{2}$
$\therefore \left(\frac{k}{2}, 0\right)$

Passes through



② $(x+4)^2 = 0$
$x + 4 = 0$
$x = -4$
$(-4, 0)$

$\because$ Solved from $(\ldots)^2$, graph touches x-axis at $(-4, 0)$



④ When $x = 0$

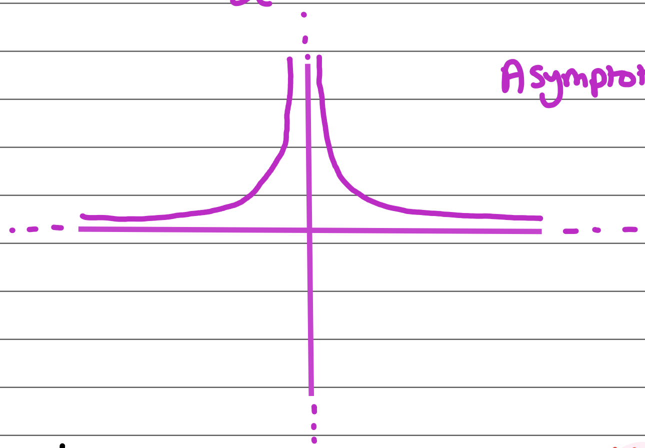
$y = (2(0) - k)(0+4)^2 = -16k$

$\therefore (0, -16k)$

P 7 2 1 3 7 A 0 1 4 2 8

ii) $y = \frac{K}{x^2}$

graph of $y = \frac{1}{x^2}$ is:



if $y = \frac{1}{x^2}$ is $y = f(x)$, $y = \frac{K}{x^2}$ is $y = Kf(x)$

$\therefore$ graph of $y = \frac{K}{x^2}$ is the graph of $y = \frac{1}{x^2}$ stretched vertically by factor K.

$\because y = K \times 0 = 0$

Asymptotes are still: $y = 0$, $x = 0$

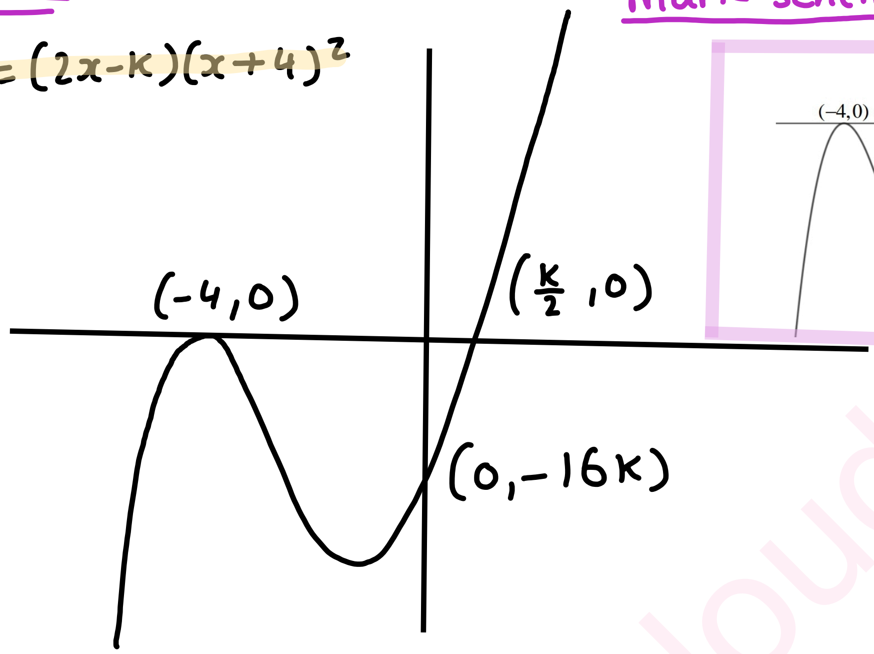
$\because$ only y-values affected

graph DOES NOT intersect coordinate axis.

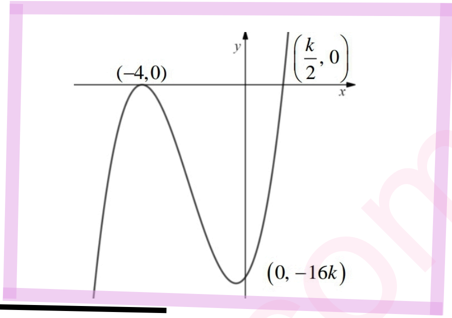
Question 6 continued

ANSWERS:

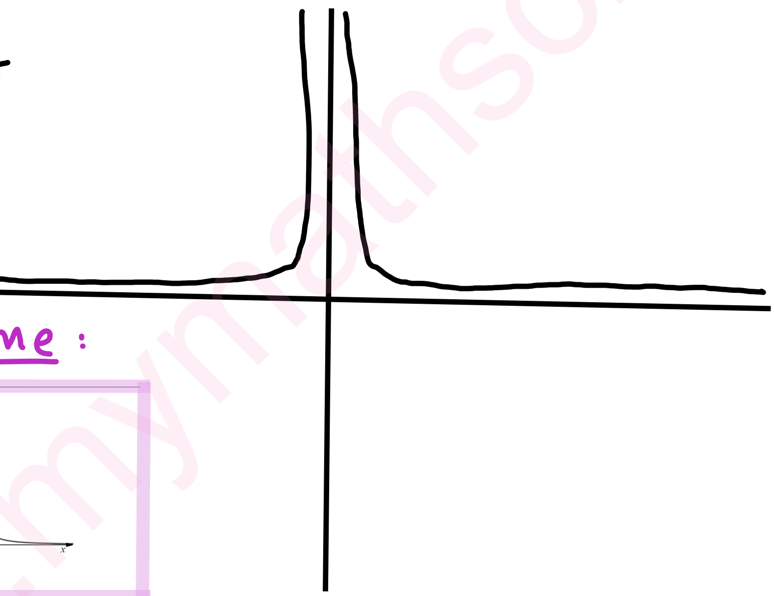
a) i) $y = (2x-k)(x+4)^2$



Mark scheme:



ii) $y = \frac{k}{x^2}$



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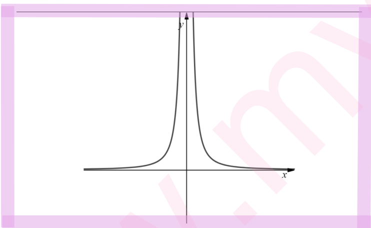
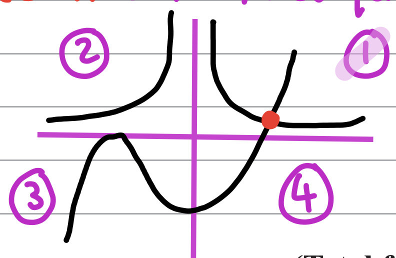


b) 1 root ∵ one intersection

intersection is in First quadrat:



(Total for Question 6 is 6 marks)

7.

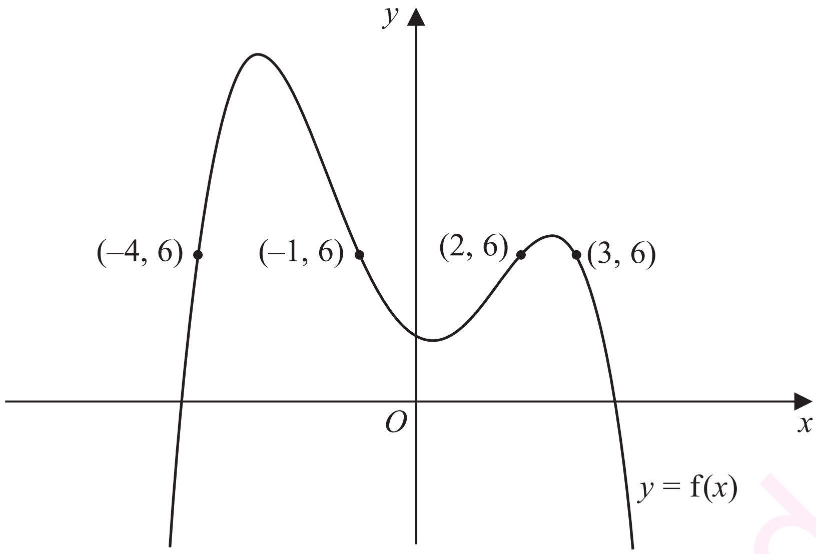


Figure 1

Figure 1 shows the curve with equation $y = \mathrm{f}(x)$.

The points $P(-4, 6)$, $Q(-1, 6)$, $R(2, 6)$ and $S(3, 6)$ lie on the curve.

(a) Using Figure 1, find the range of values of x for which

$$\mathrm{f}(x) < 6$$

(3)

(b) State the largest solution of the equation

$$\mathrm{f}(2x) = 6$$

(1)

(c) (i) Sketch the curve with equation $y = \mathrm{f}(-x)$.

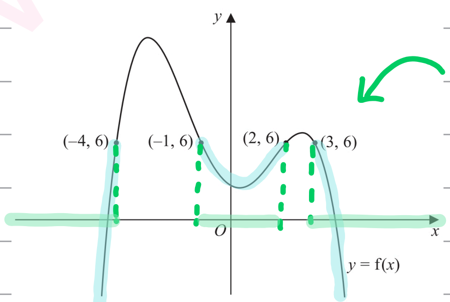
On your sketch, state the coordinates of the points to which P, Q, R and S are transformed.

(ii) Hence find the set of values of x for which

$$\mathrm{f}(-x) \geqslant 6 \text{ and } x < 0$$

(4)

a) $f(x) < 6$ means all y-values of graph below 6.



range of x will be

NOT $\geqslant$ $\because f(x) < 6$ so NOT including when $y = 6$

$\therefore$ $x < -4$, $-1 < x < 2$, $x > 3$

P 7 2 1 3 7 A 0 1 6 2 8

Question 7 continued

b) $f(2x) = 6$

largest solution of $f(x) = 6$ is 3.

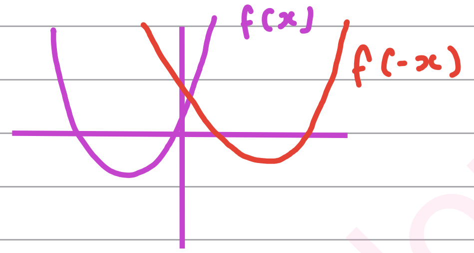
$f(2x)$ is horizontal squash by factor 2 (multiply by ½)

∴ for $f(2x) = 6$, largest solution is $3 \times \frac{1}{2} = 1.5$

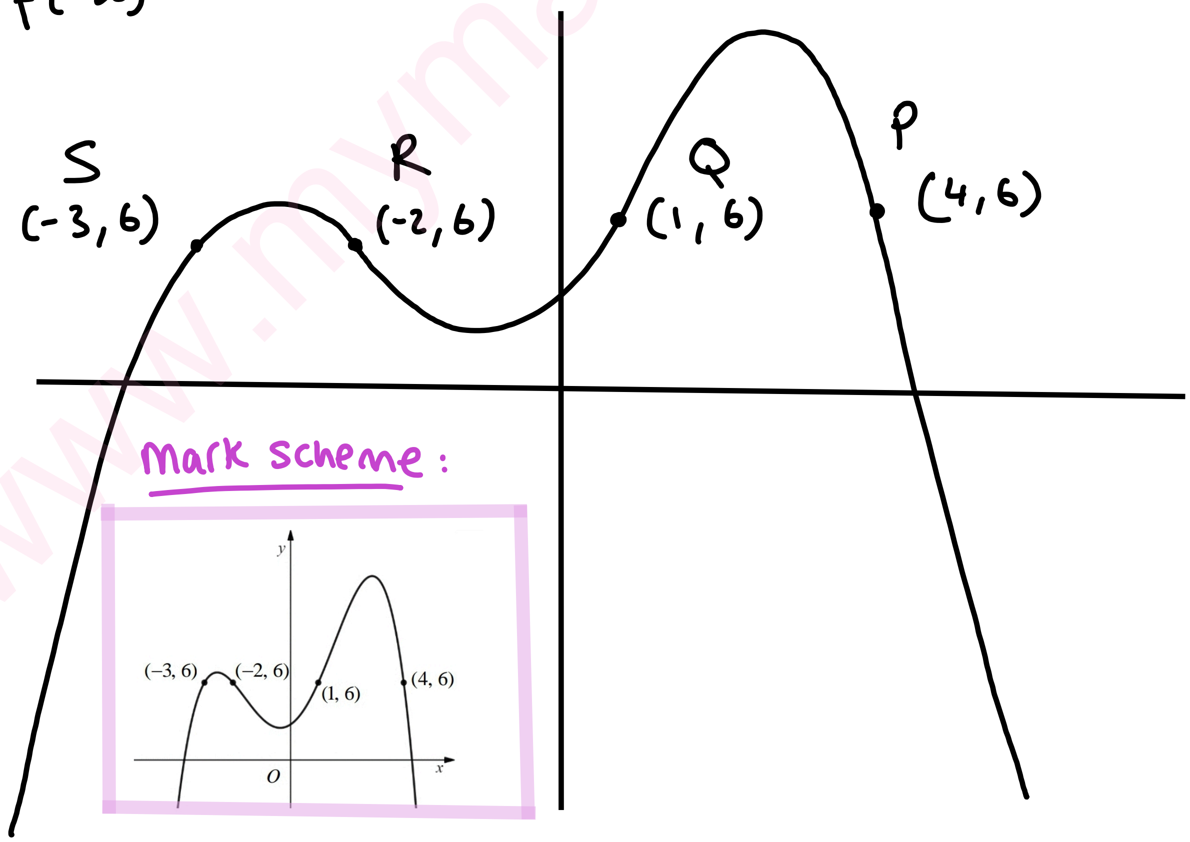
∴ largest solution is 1.5

c) i) $y = f(-x)$ is reflection by y-axis

e.g.



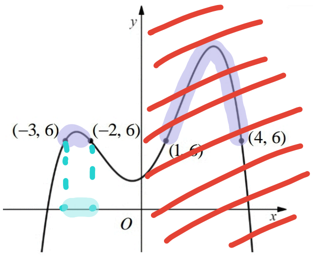
i) $f(-x)$



Question 7 continued

ii) $f(-x) \geqslant 6$ & $x < 0$

$f(-x) \geqslant 6$ is all y-values greater than or equal to 6



to find range of x:

→ $x<0$ so DO NOT INCLUDE positive x-axis

range of x will be

$\therefore -3 \leqslant x \leqslant -2$

↑ $\leqslant$ & not $<$ $\because f(x) \geqslant 6$

so including when equal to 6.

Question 7 continued

(Total for Question 7 is 8 marks)

8.

Diagram **NOT** to scale

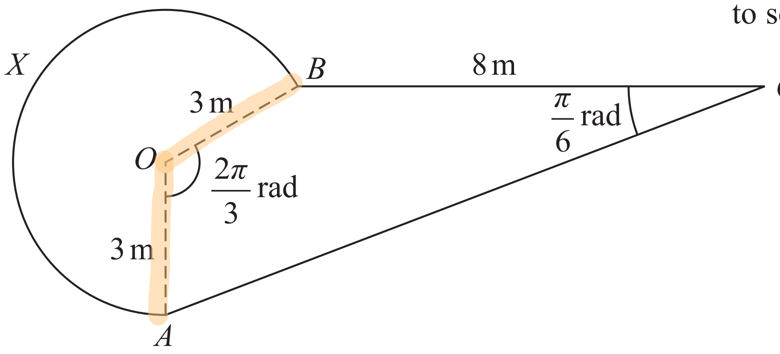


Figure 2

Figure 2 shows the plan view of a design for a pond.

The design consists of a sector *AOBX* of a circle centre *O* joined to a quadrilateral *AOBC*.

- $BC = 8\,\text{m}$
- $OA = OB = 3\,\text{m}$
- angle AOB is $\frac{2\pi}{3}$ radians
- angle BCA is $\frac{\pi}{6}$ radians

(a) Calculate (i) the exact area of the sector *AOBX*,

(ii) the exact perimeter of the sector *AOBX*.

(5)

(b) Calculate the exact area of the triangle *AOB*.

(2)

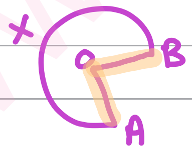
(c) Show that the length *AB* is $3\sqrt{3}$ m.

(2)

(d) Find the total surface area of the pond. Give your answer in m^2 correct to 2 significant figures.

(5)

a) i) Area of Sector : $A = \frac{1}{2}r^2\theta$



radius = OA = OB = 3m

angle in a circle (360°)

θ = obtuse $\angle AOB = 2\pi -$ acute $\angle AOB = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$

$A = \frac{1}{2}(3)^2\left(\frac{4\pi}{3}\right) = 6\pi$

$\therefore$ Area = $6\pi\ \text{m}^2$

ii) Perimeter AOBX = OA + OB + AXB = 3 + 3 + AXB

P 7 2 1 3 7 A 0 2 0 2 8

Question 8 continued

To find length of arc AXB: $S = r\theta$

radius $= 3m$

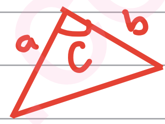
$\theta = 2\pi - \frac{3\pi}{2} = \frac{4\pi}{3}$ rad

$S = 3 \times \frac{4\pi}{3} = 4\pi\ m$ $\therefore$ AXB $= 4\pi$

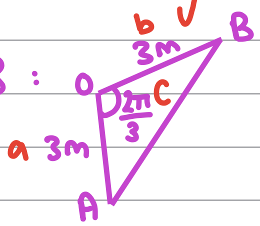
Perimeter $= 6 + AXB = 6 + 4\pi$

$\therefore$ Perimeter $= (4\pi + 6)\ m$

b) Area of a triangle : $A = \frac{1}{2}ab\sin C$ where



Triangle AOB :



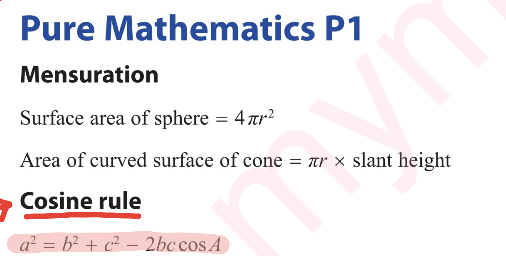
$\therefore A = \frac{1}{2} \times 3 \times 3 \times \sin\frac{2\pi}{3}$

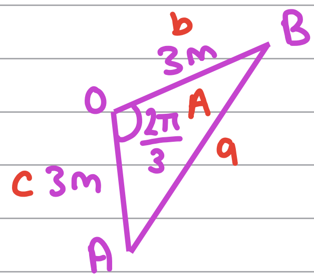
$= \frac{9}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{9\sqrt{3}}{4}$

$\therefore$ Area $= \frac{9\sqrt{3}}{4}\ m^2$

c) Use cosine rule to find length AB.





$a^2 = b^2 + c^2 - 2bc\cos A$

$AB^2 = 3^2 + 3^2 - 2(3)(3)\cos\frac{2\pi}{3}$

$AB^2 = 9 + 9 - (-9) = 9 + 9 + 9$

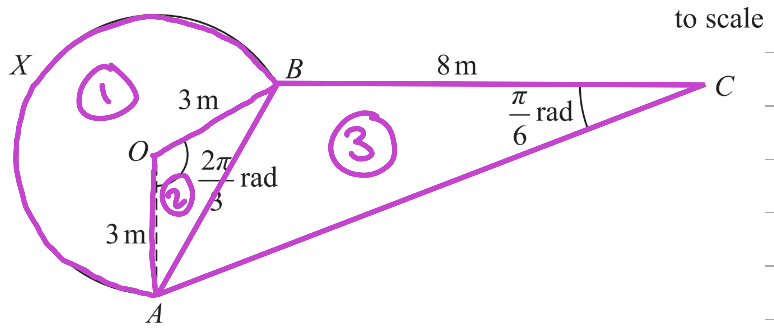
$AB^2 = 27$

$AB = \sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

$\therefore AB = 3\sqrt{3}\ m$

Question 8 continued

d) To find total surface area, split pond into 3 shapes

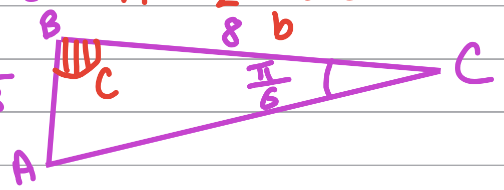


Area = ① + ② + ③

①: $6\pi\ m^2$ from part a)i)

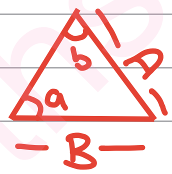
②: $\frac{9\sqrt{3}}{4}\ m^2$ from part (b)

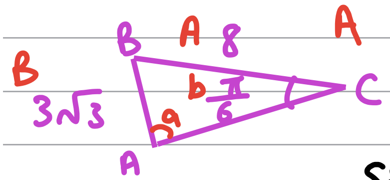
To find 3: $A = \frac{1}{2} ab \sin C$



To find C (<ABC)

Sine rule : $\frac{\sin a}{A} = \frac{\sin b}{B}$





$$\frac{\sin \angle BAC}{8} = \frac{\sin \frac{\pi}{6}}{3\sqrt{3}}$$

$$\sin \angle BAC = \frac{8 \sin \pi/6}{3\sqrt{3}} = \frac{4}{3\sqrt{3}}$$

$$\angle BAC = \sin^{-1}\left(\frac{4}{3\sqrt{3}}\right) = 0.87852\ldots \approx 0.8785$$

$$\angle ABC = \pi - \pi/6 - 0.8785 = 1.73949\ldots \approx 1.739$$

↑ total angle in a triangle (π rad = 180°)

Area of triangle ABC : $A = \frac{1}{2} \times 3\sqrt{3} \times 8 \times \sin 1.739 = 20.49127\ldots$
≈ 20.491

$\therefore$ Area = ① + ② + ③ $= 6\pi + \frac{9\sqrt{3}}{4} + 20.491$

$= 43.23767\ldots$

Question 8 continued

Surface Area of pond = 43 m^2 (2 sig fig)

(Total for Question 8 is 14 marks)

9.

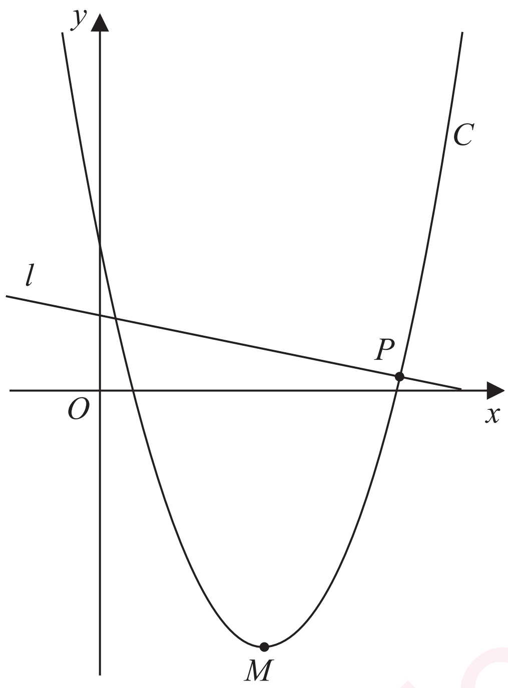


Figure 3

Figure 3 shows a sketch of the curve C with equation

$$y = \frac{1}{2}x^2 - 10x + 22$$

(a) Write $\frac{1}{2}x^2 - 10x + 22$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point M is the minimum turning point of C, as shown in Figure 3.

(b) Deduce the coordinates of M

(2)

The line l is the normal to C at the point P, as shown in Figure 3.

Given that l has equation $y = k - \frac{1}{8}x$, where k is a constant,

(c) (i) find the coordinates of P

(ii) find the value of k

(6)

Question 9 continues on the next page

Question 9 continued

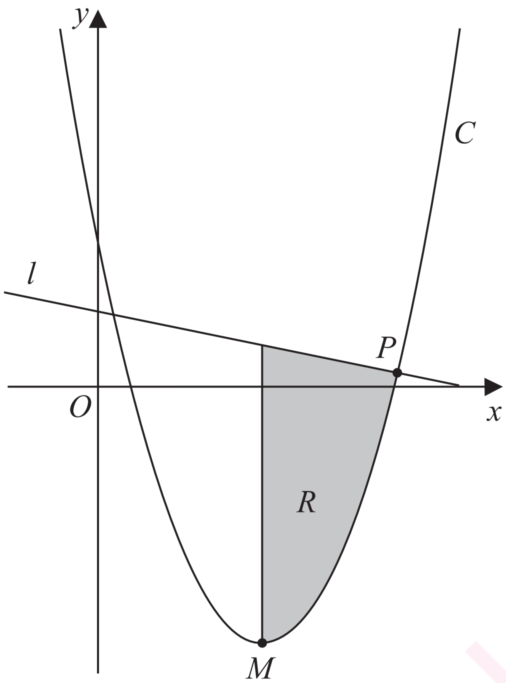


Figure 4

Figure 4 is a copy of Figure 3. The finite region R, shown shaded in Figure 4, is bounded by l, C and the line through M parallel to the y-axis.

(d) Identify the inequalities that define R.

(3)

a) Completing the square:

if $y = x^2 + bx + c$

$y = \left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$

① factorise $y = \frac{1}{2}x^2 - 10x + 22$

$y = \frac{1}{2}(x^2 - 20x + 44)$

② $y = \frac{1}{2}\left(\left(x + \left(\frac{-20}{2}\right)\right)^2 + 44 - \left(-\frac{20}{2}\right)^2\right)$

$y = \frac{1}{2}\left((x - 10)^2 - 56\right)$

$y = \frac{1}{2}(x-10)^2 + \frac{1}{2}(-56)$

$\therefore \ y = \frac{1}{2}(x-10)^2 - 28$ $\quad a = \frac{1}{2} \quad b = -10 \quad c = -28$

Question 9 continued

b) to find coordinates of minimum point:

$$y = \underbrace{\left(x + \tfrac{b}{2}\right)}_{\text{inverse is } x\text{-coordinate}}^2 + \underbrace{c - \left(\tfrac{b}{2}\right)^2}_{y\text{-coordinate}}$$

Explanation: when $y = c - (b/2)^2$: $c - (b/2)^2 = (x + b/2)^2 + c - (b/2)^2$

$0 = (x + b/2)^2$

$\therefore x = -b/2$

when $x = -b/2$: $y = (-b/2 + b/2)^2 + c - (b/2)^2$

$\therefore y = c - (b/2)^2$

$$y = \tfrac{1}{2}\underbrace{(x-10)}_{x - 10 = 0}^2 \underbrace{-28}_{y \text{ coordinate}}$$

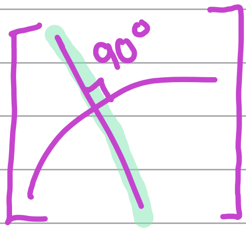
$x - 10 = 0$

$\underbrace{x = 10}_{x \text{ coordinate}}$

$\therefore M(10, -28)$

c) i) $y = k - \frac{1}{8}x$ in form $y = mx + c$: $y = -\frac{1}{8}x + c$

normal is perpendicular to curve



$\therefore$ we find gradient of curve (M_c) at point P using perpendicular gradient rule $M_{normal} \times M_{curve} = -1$

$-1/8 \times M_c = -1$

$\therefore M_c = 8$

We then find the radient function dy/dx of curve C.

$C: y = \frac{1}{2}x^2 - 10x^1 + 22x^0$ $\therefore x^0 = 1$

$dy/dx = 2\left(\frac{1}{2}x^{2-1}\right) + 1\left(-10x^{1-1}\right) + 0\left(22x^{0-1}\right)$

$= x - 10$

Question 9 continued

Equate gradient function dy/dx to the gradient of curve C at P $M_c = 8$. Then solve for x.

$$\left.\frac{dy}{dx}\right|_{x=?} = x - 10 = 8$$

$$x - 10 = 8 \quad (+10)$$

$$x = 18$$

$\therefore$ x-coordinate of P is 18

Substitute $x = 18$ into equation of curve C or line L to find y-coordinate of P.

$$C: \; y = \frac{1}{2}x^2 - 10x + 22$$

$$y = \frac{1}{2}(18)^2 - 10(18) + 22 = 4$$

$\therefore$ y-coordinate of P is 4

$$\therefore P(18, 4)$$

ii) Line l: $y = k - \frac{1}{8}x$

Substitute values of point $P(18,4)$ into equation & solve for k

$$4 = k - \frac{1}{8}(18)$$

$$4 = k - \frac{18}{8}$$

$$4 = k - \frac{9}{4} \quad \left(+\frac{9}{4}\right)$$

$$\frac{25}{4} = k$$

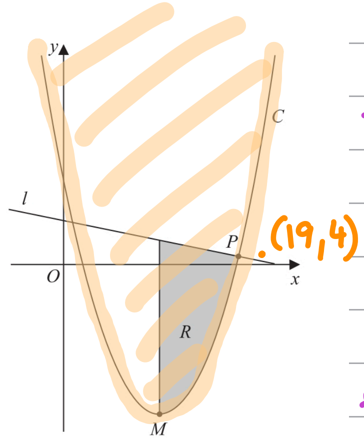
$$\therefore k = \frac{25}{4}$$

d) 3 inequalities

working in next page

Question 9 continued

1



Curve C: $y = \frac{1}{2}x^2 - 10x + 22$

To find inequality, select a point OUTSIDE valid region, then make equation FALSE.

Point chosen is (19, 4)

$4 = \frac{1}{2}(19)^2 - 10(19) + 22$

$4 = 12.5$

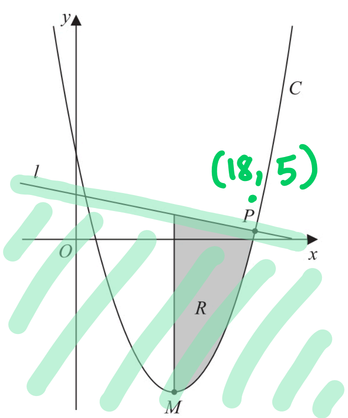
to make it FALSE: $4 \geqslant 12.5$

$\therefore y \geqslant \frac{1}{2}x^2 - 10x + 22$

* ↳ $\geqslant$ NOT $>$ ∵ line is solid, not dashed

[Solid dashed]

2



Line l: $y = \frac{25}{4} - \frac{1}{8}x$

To find inequality, select a point OUTSIDE valid region, then make equation FALSE.

Point chosen is (18, 5).

$5 = \frac{25}{4} - \frac{1}{8}(18)$

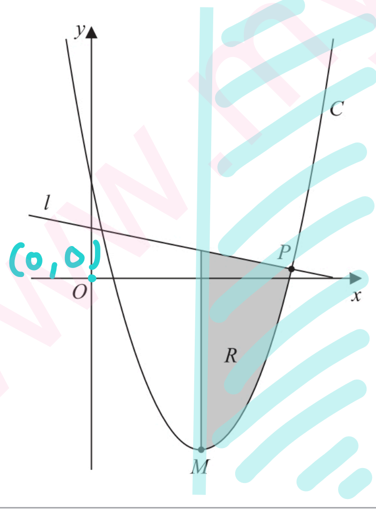
$5 = 4$

to make it FALSE: $5 \leqslant 4$

$\therefore y \leqslant \frac{25}{4} - \frac{1}{8}x$

*

3



$x = 10$

we obtain this equation by knowing that M is (10, −28) & that line through M is parallel to y-axis

To find inequality, select a point OUTSIDE valid region, then make equation FALSE.

Point chosen is (0, 0).

$0 = 10$

to make it FALSE: $0 \geqslant 10$

$\therefore x \geqslant 10$

*

$\therefore y \geqslant \frac{1}{2}x^2 - 10x + 22$ $y \leqslant \frac{25}{4} - \frac{1}{8}x$ $x \geqslant 10$

(Total for Question 9 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS